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# SPECTRUM SPREADING EFFECT OF BANDWIDTH EXPANSION ON SPECTRAL EFFICIENCY OF CELLULAR SYSTEMS

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**Abstract-** Based on a six co-channel interference model, this paper examines the effect of bandwidth expansion/spectrum spreading beyond the baseband, on the spectral efficiency of wideband and spread spectrum cellular systems. The improvement in the signal to noise ratio/protection ratio as a result of bandwidth expansion is considered. The advantage in the protection ratio combined with the resulting expanded channel spacing are weighed using an equation devised by the authors to deduce whether or not such expansion is spectrally justifiable. Various analogue and digital systems are considered. The Shannon model is treated as an ideal modulation system and SSB is taken as a reference where bandwidth expansion is not employed.

## 1. Introduction

In general, cellular systems can be classified as either narrowband or wideband systems, and advocates of each claim a superior spectral efficiency over the other. Narrowband systems use the smallest possible channel spacing to economise on spectrum in order to achieve a high spectral efficiency. On the other hand, wideband systems deliberately expand the channel spacing beyond the baseband, in order to attain a better immunity against co-channel interference. Thus, leading to a smaller re-use distance and hence achieving a higher spectral efficiency.

The spectral efficiency of a cellular land mobile radio system measured in terms of Channels/MHz.Km<sup>2</sup>, can be mathematically interpreted as follows [1]:

$$\eta_m = \frac{3}{B_c \cdot \left(\frac{D}{R}\right)^2 \cdot A} \quad (1)$$

where:

$\eta_m$ : Modulation efficiency in Channels/MHz.Km<sup>2</sup>.

$B_c$ : Voice channel spacing in MHz.

$D$ : Minimum co-channel cell separation.

$R$ : Cell radius.

$A$ : Cell area in Km<sup>2</sup>.

Considering a six co-channel interference model, Equation (1) becomes [2]:

$$\eta_m = \frac{3}{B_c \cdot (6 \cdot a) \left(\frac{2}{\alpha}\right) \cdot A} \quad (2)$$

where 'a' is the co-channel protection ratio.

Equation (2) shows that the spectral efficiency is inversely proportional to the channel spacing. Hence, the spectral efficiency of a cellular system can be increased by using the narrowest possible channel spacing; an argument for narrowband cellular systems. Conversely, Equation (2) shows

that the spectral efficiency is inversely proportional to some power  $2/\alpha$  of the protection ratio, where  $3 < \alpha < 4$ . Expanding the channel spacing can actually reduce the protection ratio needed for a given voice quality and hence increases the spectral efficiency, an argument for wideband cellular systems. It is obvious that a trade off between channel spacing and protection ratio is needed in order to arrive at the highest possible spectral efficiency.

The effect of bandwidth expansion, characteristic of wideband systems, on the spectral efficiency is examined within the cellular environment. Our criterion is to consider the improvement in the signal to noise ratio as a result of using a particular modulation technique, when the baseband channel spacing is expanded by a factor of  $x$ , from  $B_b$  to  $B_c = xB_b$ . The improvement in the signal to noise ratio results in a smaller protection ratio. The advantage in the protection ratio combined with the resulting expanded channel spacing is compared to that dictated by Equation (2). Hence, deducing whether or not such an expansion is spectrally justifiable. The following systems are considered: spread spectrum, WBFM, AM, DSB and M-ary digital modulation schemes. The Shannon model is considered as an ideal system and SSB is taken as a reference where bandwidth expansion is not employed.

## 2. Six Interferers Cellular Model

From Equation (2), within a cellular environment, any modulation technique which employs a bandwidth expansion of  $x$  times requires an improvement in the protection ratio by at least  $x^{\alpha/2}$  times, for such bandwidth expansion to be spectrally feasible. In dB's, the required improvement in the protection ratio is:

$$5\alpha \log(x) \quad (3)$$

For  $\alpha=4$ :

$$20 \log(x) \quad (4)$$

Thus, 6 dB improvement in the protection ratio is required per doubling the bandwidth.

## 3. Spread Spectrum

In spread spectrum systems, the baseband bandwidth is expanded /spread in order to improve the signal to noise ratio.

### Quantification

Let  $S$  = Signal power in Watts.  
And  $\eta/2$  = White noise in Watts/Hz.

Before spreading:

$$\begin{aligned} \text{S/N} &= \text{S}/\eta B_b & (5) \\ \text{Or } \text{S/N} &= 10 \cdot \log(\text{S}/\eta B_b) \text{ dB} & (6) \end{aligned}$$

After spreading:

Replacing  $B_b$  with  $B_c = xB_b$ :

$$\begin{aligned} \text{S/N} &= \text{S}/(\eta x B_b) \\ \therefore \text{S/N} &= 10 \cdot \log(\text{S}/\eta B_b) - 10 \cdot \log(x) \text{ dB} & (7) \end{aligned}$$

Equation (7) shows that expanding the bandwidth by a factor of  $x$  using spread spectrum techniques, results in a signal to noise improvement of  $10 \cdot \log(x)$  (i.e. 3dB per doubling the bandwidth). This is less than the  $\{20 \cdot \log(x)\}$  improvement dictated by the six interferers cellular model. Hence, expanding the channel spacing using spread spectrum techniques is not spectrally justifiable in a cellular environment. Spread spectrum is feasible for  $\alpha=2$ , which is the case in free space communication.

#### 4. WBFM

In wideband FM systems, bandwidth is traded for a better signal to noise ratio. Using a wider bandwidth will achieve the same voice quality at a lower signal to noise ratio. The signal to noise ratio of an FM signal after demodulation is given by the following equation [3]:

$$\text{S/N} = 3\beta^2 \gamma P_m \quad (8)$$

Where  $\gamma$  is the signal to noise ratio before bandwidth expansion. For WBFM,  $\beta > 1$  and  $B_T = 2\beta f_m$ . For our purpose,  $B_T = B_c$ ,  $f_m = B_b$  and  $B_c = xB_b$ . Also,  $P_m = 1/2$ , for a sine wave message signal.

$$\begin{aligned} \text{S/N} &= (3/8)x^2 \gamma \\ \text{In dB: } \text{S/N} &= 20 \cdot \log(x) - 4.3 + \gamma_{dB} & (9) \end{aligned}$$

Assuming that  $\gamma$  is so high that the discriminator is working in its linear region (above threshold). Equation (9) shows that expanding the bandwidth  $x$  times using WBFM results in a signal to noise advantage of  $\{20 \cdot \log(x) - 4.3\}$  dB. However, this is less than the  $\{20 \cdot \log(x)\}$  dB improvement dictated by the six interferers cellular model. Hence, expanding the channel spacing using WBFM technique may not be spectrally efficient within a cellular environment. Equating  $\{20 \cdot \log(x) - 4.3\}$  and  $\{5\alpha \cdot \log(x)\}$ , it can be shown that WBFM is spectrally feasible for values of  $\alpha$  less than 3 (e.g.  $x=7.24$  at  $\alpha=3$ ).

#### 5. Shannon Limit

Using the Shannon model to represent an ideal communication system, the effect of bandwidth expansion on the spectral efficiency within a cellular system, is tested. According to Shannon, the channel capacity  $C$  in bits per seconds is given by:

$$C = B \log_2 \{1 + (\text{S/N})\} \quad (10)$$

where  $B$  is the system bandwidth in Hertz.

According to Shannon, it is not possible by any encoding method to send at a higher rate and have an arbitrary low frequency error. For our purpose, we need to examine the effect of bandwidth expansion on the system's signal to noise ratio. According to Equation (10), the same information can be sent using a baseband channel spacing  $B_b$  and a corresponding signal to noise ratio  $\gamma$ ; or using a much wider bandwidth  $B_c = xB_b$  but a lower signal to noise ratio  $\text{S/N}$ . Equating the two cases:

$$\begin{aligned} B_b \log_2 (1 + \gamma) &= B_c \log_2 \{1 + (\text{S/N})\} \\ \therefore B_b \log_2 (1 + \gamma) &= x B_b \log_2 \{1 + (\text{S/N})\} \end{aligned}$$

Hence:

$$(1 + \gamma) = \{1 + (\text{S/N})\}^x \quad (11)$$

For  $\gamma > 1$  and  $\text{S/N} > 1$ :

$$\begin{aligned} \text{S/N} &= \gamma^{1/x} \\ \text{In dB: } \text{S/N} &= (1/x) \gamma_{dB} & (12) \end{aligned}$$

Equation (12) shows that expanding the bandwidth by  $x$  times using an ideal communication system results in reducing the required signal to noise requirement by  $x$  times, when  $\text{S/N}$  is expressed in dB. In other words, doubling the bandwidth will result in halving the signal to noise ratio, expressed in dB.

#### 6. DSB, AM and SSB

DSB and AM modulation systems do not employ bandwidth expansion. Nevertheless, they both use twice the bandwidth of that of a baseband message signal with no advantage in terms of the signal to noise ratio. On the contrary, an AM system can be shown to a signal to noise deterioration of about 5 dB, compared with baseband transmission.

In theory, SSB systems use the same bandwidth and have the same signal to noise ratio of a baseband message signal.

#### 7. Power-Bandwidth Exchange in Digital Systems and Spectral Efficiency

M-ary digital modulation schemes are often used whenever bandwidth needs to be conserved at the expense of power and vice versa. In this section, the effect of conserving bandwidth on the spectral efficiency within cellular systems is tested using the same criterion as before. Namely, an M-ary scheme which employs a bandwidth compression of  $x$  times, is considered spectrally efficient if no more than  $\{20 \cdot \log(x)\}$  dB deterioration in the  $\text{S/N}$  or protection ratio is caused. That is 6 dB per halving the bandwidth. The objective protection ratio values for various digital modulation techniques shown in Figure 1 will be used. Such objective protection ratio values are taken from a selection of papers [4], [5] and [6].

##### 7.1 AM Digital Techniques

From Figure 1, it can be seen that using QAM instead of coherent ASK can theoretically double the modulation speed with no change in the objective protection ratio. Hence, leading to a more spectrally efficient digital cellular system. However, moving from QAM to 16-QAM will result in a loss in spectral

Type	Digital Modulation and Detection	Theoretical		Practical		Objective Protection Ratio (dB) $BER=10^{-2}$ (Fading)
		Speed (b/s/Hz)	Equivalent $B_c$ (kHz)	Speed (b/s/Hz)	Equivalent $B_c$ (kHz)	
AM	ASK-Envelope	1	16	<0.8	>20	19
	ASK-C	1	16	0.8	20	17
	QAM	2	8	1.7	9.4	17
	QPR	2.5	6.4	2.25	7.1	20.5
	16-QAM	4	4	3.0	5.3	27.5
FM	FSK (h=1)-NC	1	16	0.8	20	20
	CP-FSK (h=0.7)-NC	1	16	1.0	16	18
	CP-FSK (h=0.7)-C	1	16	1.0	16	13
	MSK (h=0.5)-C	2	8	1.9	8.4	17
	MSK (h=0.5)-DE	2	8	1.9	8.4	20
PM	BPSK-C	1	16	0.8	20	14
	DE-BPSK	1	16	0.8	20	17
	DPSK	1	16	0.8	20	17
	DQPSK	2	8	1.8	8.9	23
	QPSK	2	8	1.9	8.4	16.5
	8-PSK	3	5.3	2.6	6.2	21.3
	16-PSK	4	4	2.9	5.5	27
AM/PM	4-APK	2	8	<2	>8	16.5
	8-APK	3	5.3	2.8	5.7	19.8
	16-APK	4	4	3.3	4.8	24

C: Coherent NC: Noncoherent DE: Differential Encoding

Fig. 1 Theoretical and Practical Equivalent Channel Spacing and Objective Protection Ratio for Various Digital Modulation Techniques

efficiency. This is because the objective protection ratio drops by about 10 dB, while only  $20\log(3/1.7) = 4.9$  dB drop is allowed to maintain the spectral efficiency. In this respect, QAM is the most spectrally efficient AM digital scheme, amongst those shown in the table.

## 7.2 FM Digital Techniques

Using the same argument above, it is spectrally feasible to move from FSK to MSK. MSK can be shown to be the most efficient FM digital scheme, amongst those shown in the table.

## 7.3 PM Digital Techniques

Considering the practical modulation speeds and the objective protection ratios shown in Figure 1, the spectral efficiency increases when moving from BPSK to QPSK. However, moving to 8-PSK and 16-PSK will result in a lower spectral efficiency.

## 7.4 Hybrid AM/PM Digital Techniques

APK is an example of this category. Figure 1 shows that moving from 4-APK to 8-APK does not in fact increase the spectral efficiency. A loss in spectral efficiency will result if 16-APK is used instead of 8-APK. In this category, 4-APK is spectrally the optimum scheme to employ.

## 8. Comparison of Bandwidth Expansion Methods and Conclusions

Figure 2 shows a tabulated comparison of various modulation systems employing bandwidth expansion. The six interferers model is included as a reference to judge the spectral viability of bandwidth expansion employed by various systems. It is obvious that employing bandwidth expansion using spread spectrum and WBFM is not justifiable within a cellular environment. This is because the protection ratio advantage as a result of bandwidth expansion is less than that dictated by the cellular model, hence, an inferior spectral efficiency is the outcome. The same applies to DSB and AM systems. Only the ideal system predicted by Shannon shows a considerable increase in the spectral efficiency as a result of bandwidth expansion. SSB shows an interesting result, since it is a special case of the ideal system when no expansion is employed (i.e.  $x=1$ ).

Figure 3 shows a graphical representation of bandwidth expansion on the protection ratio for various modulation systems, for  $x=10$ . The six interferers cellular model is included as a reference, such that, systems which fall below the cellular model will have a spectral efficiency disadvantage by employing bandwidth expansion. That includes spread spectrum and WBFM systems. Only the ideal system predicted by Shannon may benefit from bandwidth expansion as far as spectral efficiency is concerned. Other systems which do not employ bandwidth expansion such as DSB, AM and SSB are also included in Figure 6.3. SSB uses the minimum bandwidth possible (i.e. Baseband bandwidth) with no penalties in terms of signal to noise ratio, hence, spectrally more efficient than DSB and AM systems. Evaluation of the spectral efficiency of practical cellular systems will test the above results.

For digital modulation systems, it is only spectrally feasible to move from 2-levels to 4-levels schemes. In this respect, QAM, QPSK and 4-APK prove to have a superior spectral efficiency in a cellular environment. Using higher level digital schemes is not spectrally feasible in a cellular system.

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Modulation System	Bandwidth Expansion	S/N	Protection Ratio Advantage (dB)
Cellular, Six Interferers Model	x	$\frac{x^2 \gamma}{\alpha = 4}$	$20 \log(x)$
Shannon-Hartley	x	$\gamma^x$	$x \alpha_{dB}$
Spread Spectrum	$x \gg 1$	$x \gamma$	$10 \log(x)$
WBFM	$x \gg 1$	$\frac{3}{8} x^2 \gamma$	$20 \log(x) - 4.3$
DSB	$x = 2$	$\gamma$	—
AM	$x = 2$	$\frac{\gamma}{3}$	- 4.8
SSB (Baseband)	$x = 1$	$\gamma$	—

Fig. 2 Effect of Bandwidth Expansion on Modulation Efficiency

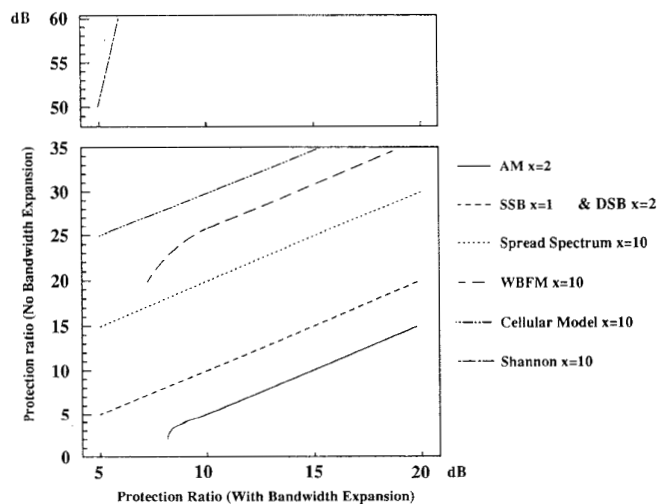


Fig. 3 Effect of Bandwidth Expansion on Protection Ratio